Gender Differences in How Students Solve the Most Difficult to Retrieve Single-Digit Addition Problems

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Despite curriculum expectations, many students, including a disproportionate number of girls, do not 'just know' (retrieve) single-digit addition facts by Year 3. The current study employed structured interviews to explore which strategies Year 3/4 students (n = 166) used when solving more difficult addition combinations. Results revealed that students preference the near-doubles strategy when the difference between the addends was one, the bridging-through-10 strategy when one of the addends was a nine, and the count-on-from-larger strategy when a derived strategy was more effortful. Moreover, whereas boys were more inclined to use derived strategies, girls were almost three times more likely to use the count-on-from-larger strategy.

In Australia, students are expected to learn their single-digit addition combinations using increasingly efficient strategies in Year 1 and Year 2, so they can fluently recall (i.e., 'just know') their addition facts by the end of Year 3 (ACARA, 2015). Moreover, this pathway towards fluent recall put forward in the Australian curriculum mathematics (Version 8.4) is consistent with learning trajectories of addition fact mastery that have been postulated in educational research (Baroody, 2006; Carpenter et al., 2015). For a problem such as 8 + 6, it might be expected that:

- a Year 1 student would count-on from the larger number (8, 9 10, 11, 12, 13, 14);
- a Year 2 student would use an efficient derived strategy, such as: near-doubles (6 + 6 + 2), bridging-through-10 (8 + 2 + 4 or 6 + 4 + 4), compensate—overshoot (6 + 10 - 2), compensate—equalise (6 + 8 = 7 + 7) and;
- a Year 3 student would simply recall that "8 plus 6 is 14", and could use this knowledge to solve more complex problems, such as recognising that 78 + 56 is equivalent to 120 and 14.

However, as many teachers observe, by the end of Year 3, large numbers of students do not 'just know' (i.e., retrieve) addition facts, or use efficient derived strategies, but continue to 'count-on'. Research confirms that over one-third of students continue to rely on accurately executed counting-based strategies into Year 3 and beyond (Gervasoni et al., 2017; Hopkins & Bayliss, 2017). Compared with students using more efficient strategies, students who continue to count-on perform more poorly on standardised mathematics assessments (Hopkins & Bayliss, 2017), and have lower levels of mental computation flexibility (Hopkins et al., 2022).

Although relying on counting-based strategies beyond the stage when it is developmentally appropriate to do so is generally considered problematic, little is known about how student strategy use varies across different types of single-digit addition problems. The purpose of the current paper is to examine how students solve the most difficult to retrieve single-digit addition problems, and to consider whether this strategy profile varies across gender.

Gender Differences in Strategy Use

There are notable differences between boys and girls in the tendency to rely on counting-based strategies compared with retrieval (Bailey et al., 2012). Carr and Jessup (1997) interviewed 58 Year 1 students on three occasions throughout the school year to examine how they solved simple addition and subtraction problems. They found that gender differences in the use of counting-based strategies vis-à-vis retrieval increased across the year. By the end of the year, girls were 1.73 times more likely to use an overt counting-based strategy than boys, and 1.79 times more likely to correctly execute such a strategy. By contrast, boys were 1.48 times more likely to attempt to use a retrieval strategy,

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and 1.58 times more likely to correctly execute a retrieval strategy. Similarly, Carr and Davis (2001) interviewed 84 Year 1 students about how they solved single-digit addition and subtraction problems and found that girls were more likely to attempt (1.83 times) and correctly execute (1.91 times) an overt counting-based strategy, whilst boys were more likely to attempt (1.75 times) and correctly execute (1.95 times) a retrieval strategy. Moreover, there is evidence that these gender differences in both preferences for using retrieval, and accuracy retrieving, persist throughout primary school (Bailey et al., 2012), and into secondary school (Hopkins & Bayliss, 2017).

Explanations as to why these gender differences exist and persist have not been explored in depth. After finding that girls were more than twice (2.26 times) as likely to be clustered into a strategy profile group defined by their propensity to accurately count-on from the larger number, Hopkins and Bayliss (2017) concluded that these differences possibly reflect girls setting "a higher threshold for determining confidence with retrieval" (p. 30). Bailey et al.'s (2012) study found these gender differences were not related to either differences in central executive function or intelligence, and instead postulated that boys' relative preference for "risk taking" in settings involving social evaluation and greater interest in competition, results in them developing a preference for using retrieval. This preference leads to more practice using retrieval-based strategies, which in turn results in superior retrieval performance for boys.

Gaps in the Research

There are two limitations to the current suite of studies on single-digit addition that present gaps in our understanding of how students solve single-digit addition problems. First, generally single-digit addition (and subtraction) problems have been considered as a single, coherent category (e.g., Carr & Jeeup, 1997; Hopkins & Bayliss, 2017), and how students perform on a subset of single-digit addition problems that possess particular characteristics, such as those problems identified as difficult to retrieve, has tended to not be an explicit focus of prior research. This means that particular phenomenon that have been observed, such as girls being more inclined to use counting-based strategies, are perhaps not as well understood as they might be. For example, it may be that the magnitude of the gender differences from the Carr and Davis (2001) and Carr and Jessup (1997) studies are either masked or amplified by the specific number facts chosen for inclusion in these studies compared with, for example, gender differences on those single-digit addition facts that students find most difficult to retrieve. Second, prior studies have often not delineated the use of efficient derived strategies from covert counting-based strategies (e.g., Carr and Davis, 2001). Moreover, even when they do make this delineation, they generally do not distinguish between the various derived strategies, such as near-doubles or bridging-through-10, but rather collapse them all into a single category (e.g., Geary et al., 1996). This is problematic if one considers that exploring different strategies for solving single-digit addition problems, and the various derived strategies in particular, is a large focus of contemporary instruction in number in the first three years of school (e.g., ACARA, 2015). The current study seeks to address these two limitations by focusing on a subset of single-digit addition problems that students find most difficult to retrieve (see Russo & Hopkins, 2022), as well as distinguishing between the various derived strategies and reporting on these separately.

The Current Study

Given that we know large numbers of students are not able to recall their single-digit addition combinations (Hopkins & Bayliss, 2017), we wondered which of these combinations students find most difficult to retrieve. Surprisingly from our perspective, the research literature relied on data that was over 80 years old to inform us on this question (see Wheeler, 1939). Moreover, this data was collected following an intervention in which the use of number sense strategies was actively discouraged, less they interfere with rote memorisation. Consequently, as part of a larger research project, we decided to investigate the issue ourselves (see Russo & Hopkins, 2022). We were interested in finding out which addition combinations students found most difficult to recall. We invited students in Years 3 and 4 to solve 36 single-digit addition problems (see Table 1) under two conditions (a strategy choice condition and a quick response condition) and used this data to create a composite measure designed to capture student difficulty retrieving addition facts. Using this composite measure, the 10 most difficult single-digit addition combinations for students to accurately recall are (in descending order): 6 + 9; 7 + 8; 7 + 9; 6 + 8; 5 + 9; 5 + 8; 6 + 7; 5 + 7; 8 + 9; 4 + 9. The current paper delves deeper into our data to explore which strategies students tended to rely on when solving these more difficult combinations under the strategy choice condition. An additional focus is to explore whether there were any notable differences between boys and girls, given research suggesting that boys are more likely to retrieve addition facts than girls, whilst girls are more likely to use counting-based strategies (Bailey et al., 2012; Carr & Davis, 2001). Our research questions include:

- Which strategies do students choose to use to solve difficult to retrieve addition combinations?
- Are there differences between boys and girls in terms of strategy choice when solving difficult to retrieve addition combinations?

Our study sits within a social cognitive perspective on how children develop computational strategies. This perspective contends that individual level factors (e.g., working memory), interactions with others (e.g., teachers) and contextual factors (e.g., problem type) coalesce to influence strategy choice. The key focus of the current paper is on how the individual level factor of gender interacts with the contextual factor of problem difficulty to shape student choice of strategy for solving addition problems.

Method

Victorian primary school students in Years 3 and 4 (n = 166; girls = 84; boys = 82) from three different schools in metropolitan Melbourne solved 36 single-digit addition problems during an individual structured interview. The problems included all single-digit addition problems where the smaller addend is presented first, plus doubles, but excluding plus zero and plus one (see Russo & Hopkins, 2022 for more information about the study methodology). During the interview, single-digit addition problems were presented on a screen using the Fact Cat program. Students were instructed to call out the answer to the problem as soon as they worked it out. Student responses were recorded by the researcher, who also asked the student “How did you do it?” and recorded their strategy. For the purposes of the current paper, student strategy choices when solving the 10 most difficult to retrieve single-digit addition combinations were subsequently coded into SPSS v. 26 for analysis (see Table 1), and delineated from the 26 easier to retrieve single-digit addition problems.

Results

Which Strategies do Students Choose to use to Solve Difficult to Retrieve Addition Combinations?

Table 1 summarises the strategies that students used to solve the 10 most difficult to retrieve single digit addition problems that are the focus of the current paper. Overall, students used a variety of strategies to solve these problems and there are some notable differences in the average frequency of each strategy across problems.

The most frequently used strategies overall were bridging-through-ten (26%), near-doubles (25%), count-on-from-larger (22%), retrieval (knew-it) (10%), compensate-overshoot (6%) and compensate-equalise (2%). Collectively, these strategies accounted for 91% of all strategy choices when answering these most difficult problems. The remaining 9% of strategy choice are explained by students either using a relatively inefficient count-on-from-the-first-number strategy, a count-all strategy, a highly idiosyncratic strategy or those who responded 'don't know' when asked how they worked it out.

Table 1

Strategy Choice for Solving Single-digit Addition Problems

Problem	Correct	Knew-it	Count-On	Near-doubles	Bridging-through-10	Compensate—overshoot	Compensate—equalise
4 + 9	92%	14%	23%	0%	40%	10%	1%
5 + 7	93%	13%	23%	33%	19%	0%	5%
5 + 8	93%	9%	32%	18%	28%	4%	1%
5 + 9	87%	8%	20%	12%	35%	15%	1%
6 + 7	91%	12%	17%	48%	14%	1%	0%
6 + 8	91%	7%	29%	23%	27%	1%	3%
6 + 9	89%	13%	23%	11%	32%	8%	1%
7 + 8	90%	7%	18%	50%	15%	2%	0%
7 + 9	90%	7%	22%	11%	34%	11%	4%
8 + 9	92%	8%	16%	43%	16%	8%	0%
Total	91%	10%	22%	25%	26%	6%	2%

In terms of different strategy profiles across problems, near-doubles was the most frequently employed strategy for problems where the difference between the addends was one (6 + 7; 7 + 8; 8 + 9) and was typically used by just under half of students to solve these problems. However, there were two other problems for which near-doubles was utilised by at least one-fifth of students: 5 + 7 (i.e., 5 + 5 + 2 or 7 + 7 - 2) and 6 + 8 (i.e., 6 + 6 + 2 or 8 + 8 - 2). By contrast, bridging-through-10 was the most frequently employed strategy when one of the addends was 9 (4 + 9; 5 + 9; 6 + 9; 7 + 9), with the exception of the problem 8 + 9, where the two addends had a difference of one, and near-doubles was used most frequently (i.e., 8 + 8 + 1 or 9 + 9 - 1). Although compensate-overshoot was not as frequently used as bridging-through-10, it also tended to only be employed for problems where one of the addends was 9.

Given that counting-on from the larger number is a strategy always available to students, it is noteworthy that there were considerable differences in how often it was used across different problems. Consider the three problems that each summed to 13: 4 + 9; 5 + 8; 6 + 7. Whereas approximately one-third of students counted-on to solve 5 + 8, less than one quarter counted-on to solve 4 + 9 and only around one-sixth counted-on to solve 6 + 7. Count-on does seem more likely to be used when a derived strategy is more effortful, specifically for those problems where the difference between the addends is at least two and where 9 is not one of the addends.

Are There Differences Between Boys and Girls in Terms of Strategy Choice When Solving Difficult to Retrieve Addition Combinations?

Differences between strategy use of boys and girls on the 10 most difficult to retrieve problems are presented in Table 2.

Table 2

Strategy Choice for Solving the 10 Most Difficult to Retrieve Single Addition Problems by Gender

	Correct	Knew-it	Count-On	Near-doubles	Bridging-through-10	Compensate—equalise	Compensate—overshoot
Boys	92%	11%	12%	31%	28%	2%	6%
Girls	90%	9%	33%	19%	24%	1%	6%

As is apparent from viewing the table, boys were more likely to use the near-doubles strategy (31% vs 19%), or to a lesser extent the bridging-through-10 strategy (28% vs 24%), on these most difficult problems whereas girls were more likely to use the count-on-from-larger strategy (33% vs 12%). Overall, boys used an efficient derived strategy on two-thirds of trials, whereas girls used an efficient derived strategy on half of trials.

Differences between boys and girls across problem type in their usage of the count-on-from-larger, near-doubles strategy and bridging-through-10 are displayed in Figures 1, 2 and 3 respectively. Figure 1 displays some stark differences in the propensity to count-on from the larger number between girls and boys. For example, whereas half of girls counted-on to solve 5 + 8, only one-seventh of boys used this strategy. Similarly, girls (37%) were four times as likely to employ the count-on-from-larger strategy to solve 5 + 7 compared with boys (9%). With regards to the near-doubles strategy (Figure 2), boys were consistently more likely to use this strategy than girls, however the differences across individual problems were somewhat less dramatic than when comparing the count-on-from-larger strategy. However, it is clear that boys were notably more likely to use near-doubles strategy regardless of the difference between the addends compared with girls. Specifically, with the exception of 4 + 9, for which the near-doubles strategy was not employed by any student, at least 15% of boys used the near-doubles strategy on each of the remaining nine problems, whereas the equivalent 'floor usage' of the near-doubles strategy for girls was only 5%. It is particularly striking that almost one-fifth of boys used the near-doubles strategy to solve the most difficult to retrieve problem in the data set, 6 + 9 (6 + 6 + 3 or 9 + 9 - 3), despite the bridging-through-10 or compensate-overshoot strategy appearing more efficient and less effortful. Regarding the bridging-through-10 strategy (Figure 3), gender differences on individual problems trend in the same general direction, with boys being more likely than girls to employ this strategy for all problems except for 6 + 8.

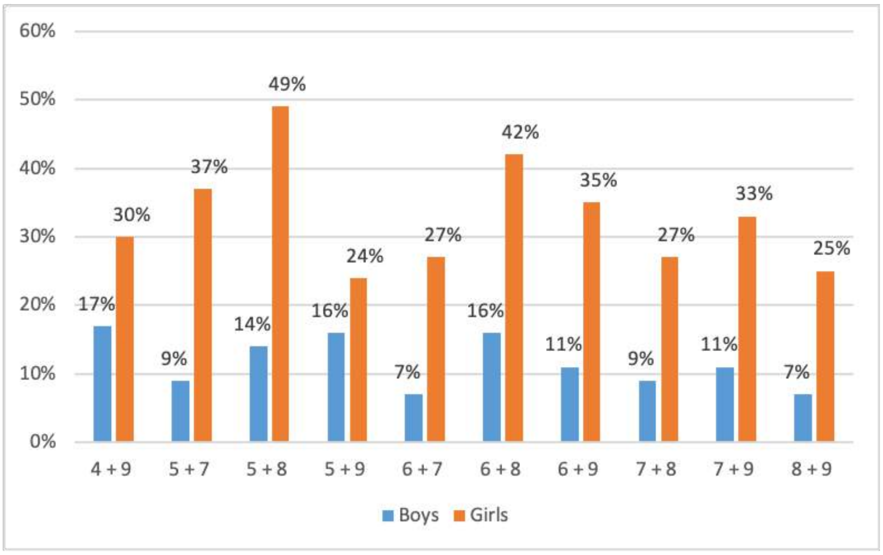

Figure 1. Count-on-from-larger strategy by problem type by gender (%).

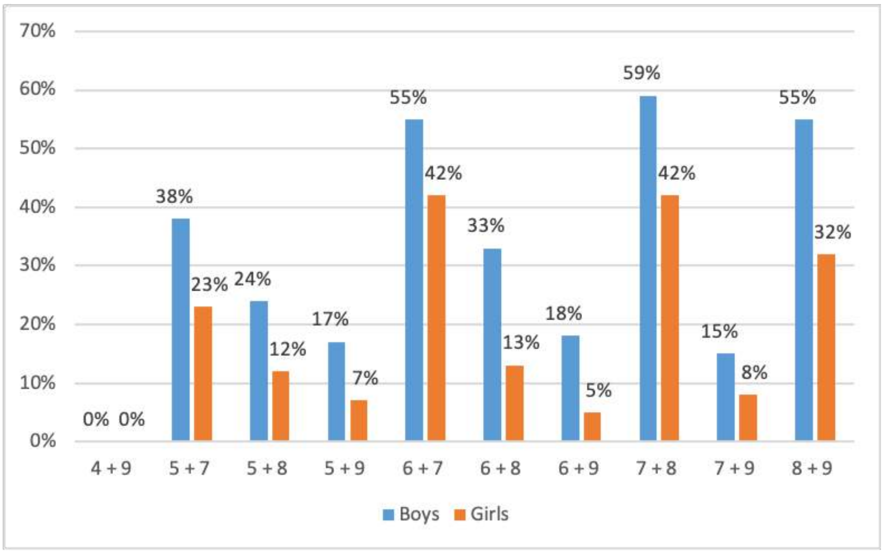

Figure 2. Near-doubles strategy by problem type by gender (%).

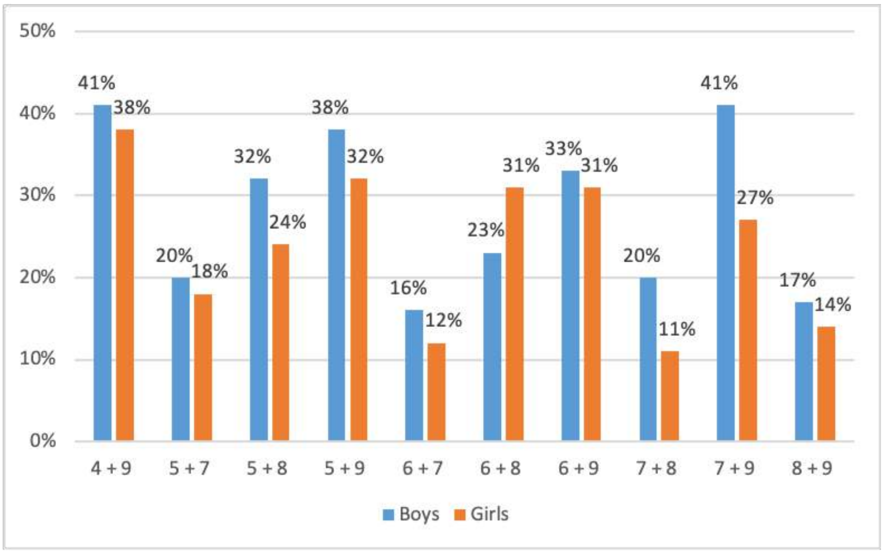

Figure 3. Bridging-through-10 strategy by problem type by gender (%).

Additional Analysis: Comparing the Propensity to Count-on Across Different Problem Types by Gender

Given these substantial gender differences in the propensity to count-on from the larger number for comparatively difficult single-digit addition problems, it is worth comparing this finding with gender differences on the remaining comparatively easy problems. It is clear from Table 3 that girls are more likely to count-on when presented with a problem from the more difficult set than when presented with a problem from the easier set, whereas problem difficulty makes relatively little difference to whether boys use the count-on strategy or not. Consequently, the ratio of girls-to-boys who use the count-on strategy is far larger for the difficult problem set (2.81 times) compared with the easier problem set (1.58 times).

Table 3

Mean Propensity to Count-On-From-Larger Number by Problem Type by Gender

	10 most difficult problems	26 easier problems	Total
Boys (total)	12%	11%	11%
Girls (total)	33%	18%	22%
Ratio (Girls/ Boys)	2.81 times	1.58 times	1.93 times

Discussion and Conclusions

Consistent with previous research (e.g., Hopkins & Bayliss, 2017; Gervasoni et al., 2017), our findings demonstrated that students use a variety of strategies to solve single-digit addition problems. Our study, however, builds on previous work by revealing how the propensity to use particular derived strategies differs substantially across problems. For example, although the bridging-through-10 strategy was potentially available for all 10 problems presented in Table 1 (given all problems summed to at least 12), the strategy was reported to be used by as few as 14% of students for the problem 6 + 7, and by as many as 40% of students for the problem 4 + 9. This suggests that students are flexible in choosing not only between counting-based strategies, derived strategies and retrieval, as has been demonstrated previously (Hopkins & Bayliss, 2017), but also in choosing amongst particular derived strategies.

Our findings are also consistent with previous research in that they revealed that girls are more inclined to use counting-based strategies than boys (Bailey et al., 2012; Carr & Davis, 2001; Carr & Jessup, 1997). However, rather than boys being more likely to use retrieval (e.g., reporting to 'just know' the answer) as has been reported previously (e.g., Carr & Davis, 2001), boys in our study were more likely to report employing derived strategies than girls, in particular the near-doubles strategy. Indeed, some boys consistently chose the near-doubles strategy when a more efficient alternative was available. Differences between our study and previous research may reflect our emphasis in seeking detailed information about the specific strategy employed from participants, and coding these strategies accordingly.

It is also notable that the magnitude of gender differences in the propensity to use counting-based strategies were larger in our study than reported previously. For example, in our study, girls were 2.81 times more likely to attempt to use the count-on-from-larger strategy than boys, notably highly than reported gender differences in the propensity to employ overt counting-based strategies in the Carr and Jessup (1997; 1.73 times) and Carr and Davis (2001; 1.83 times) studies. Further analysis revealed that these larger gender differences were likely a result of our study focussing on the 10 most difficult to retrieve problems, given that the remaining 26 easier addition problems for which data was also gathered showed substantially smaller gender differences in girls' relative use of the count-on-from-larger strategy (1.58 times more likely). The conclusion is that girls are somewhat more likely than boys to use the count-on-from-larger strategy when the problem is comparatively easy (e.g., 3 + 5), but far more likely to use the count-on-from-larger strategy when the problem is more difficult (e.g., 5 + 8).

Bailey et al.'s (2012) suggestion that boys' relative comfort (on average) with greater "risk taking" in a setting where their performance is being evaluated leads to them being less likely to favour counting-based strategies is consistent with our findings. Specifically, as problems become more difficult, more risk adverse students will be more inclined to utilise counting-based strategies to prioritise "not being incorrect". This is because, once mastered, the count-on-from-larger strategy becomes what is effectively a universal back-up strategy for solving addition problems that guarantees one arriving at the correct answer. By contrast, students who are less risk adverse will still be comparatively comfortable attempting to retrieve the answer or executing a retrieval-based strategy as problems become more difficult, even if this means a slightly higher likelihood of them making an error, in order to prioritise “being correct with minimal cognitive effort” and/ or “being correct quickly”.

Given the magnitude of the gender differences reported in our study, we would emphasise that further research is necessary to examine whether these differences are replicable across larger, more diverse samples of students. Moreover, we are reluctant to conclude that gender differences reported here are necessarily highly problematic. For example, additional data collected from the same participants suggested that, although students who relied more on accurately executing the count-on-from-larger strategy had lower levels of mental computation flexibility than students who relied more on retrieval, girls did not have statistically significantly lower levels of mental computation flexibility on average than boys. It may be that there are different pathways to mental computation flexibility that are leveraged by boys and girls, as suggested by Bailey et al. (2012). However, boys’ tendency to select and appropriately execute more efficient strategies may still have implications for gender differences in mathematical performance beyond mental computational flexibility, particularly if such differences persist throughout schooling, and warrants further research consideration. In any case, we do think that the magnitude of gender differences reported here should at least give the mathematics education research community pause when considering moving away from reporting and exploring differences between girls and boys, even in an environment where the notion of gender as a binary construct has become problematised (Hall & Norén, 2021).

References

Australian Curriculum, Assessment and Reporting Authority (2015). *The Australian curriculum: Mathematics*. ACARA. http://www.australiancurriculum.edu.au/mathematics/curriculum/f-10

Bailey, D. H., Littlefield, A., & Geary, D. C. (2012). The codevelopment of skill at and preference for use of retrieval-based processes for solving addition problems: Individual and sex differences from first to sixth grades. *Journal of Experimental Child Psychology, 113*(1), 78–92.

Baroody, A. J. (2006). Why children have difficulties mastering the basic number combinations and how to help them. *Teaching Children Mathematics*, *13*(1), 22–31. https://doi.org/10.5951/tcm.13.1.0022

Carr, M., & Davis, H. (2001). Gender differences in arithmetic strategy use: A function of skill and preference. *Contemporary Educational Psychology*, *26*(3), 330–347.

Carr, M., & Jessup, D. L. (1997). Gender differences in first-grade mathematics strategy use: Social and metacognitive influences. *Journal of Educational Psychology*, *89*(2), 318.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015). *Children’s mathematics: Cognitively guided instruction* (2nd ed.). Heinemann.

Geary, D. C., Bow-Thomas, C. C., Liu, F., & Siegler, R. S. (1996). Development of arithmetical competencies in Chinese and American children: Influence of age, language, and schooling. *Child Development*, *67*(5), 2022–2044.

Gervasoni, A., Guimelli, K., & McHugh, B. (2017). The development of addition and subtraction strategies for children in kindergarten to grade 6: Insights and implications. In A. Downton, S. Livy, & J. Hall (Eds.), 40 years on: We are still learning! *Proceedings of the 40th annual conference of the Mathematics Education Research Group of Australasia* (pp. 269–276). Melbourne: MERGA.

Hall, J., & Norén, E. (2021). Innovations in “gender issues” research in mathematics education. Mathematics *Education Research Journal, 33*(4), 787–791. https://doi.org/10.1007/s13394-021-00404-8

Hopkins, S., & Bayliss, D. (2017). The prevalence and disadvantage of min-counting in seventh grade: Problems with confidence as well as accuracy? *Mathematical Thinking and Learning*, *19*(1), 19–32. https://doi.org/10.1080/10986065.2017.1258613

Hopkins, S., Russo, J., & Siegler, R. (2022). Is counting hindering learning? An investigation into children’s proficiency with simple addition and their flexibility with mental computation strategies. Mathematical *Thinking and Learning*, *24*(1), 52–69. https://doi.org/10.1080/10986065.2020.1842968

Russo, J., & Hopkins, S. (2022). Not so Simple Addition: Comparing Student Performance and Teacher Perceptions of Retrieval. *International Journal of Science and Mathematics Education*, 1–23. https://doi.org/10.1007/s10763-022-10346-7.

Wheeler, L. R. (1939). A comparative study of the difficulty of the 100 addition combinations. *The Pedagogical Seminary and Journal of Genetic Psychology*, *54*(2), 295–312.